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TOPICAL REVIEW

Quantum radiometry

Sergey V. Polyakov[a,b] and Alan L. Migdall[a,b]*

[a]*Optical Technology Division, National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg, MD 20899-8441, USA;* [b]*Joint Quantum Institute, University of Maryland, College Park, MD 20742, USA*

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We review radiometric techniques that take advantage of photon counting and stem from the quantum laws of nature. We present a brief history of metrological experiments and review the current state of experimental quantum radiometry.

Keywords: single photon detector; metrology; high accuracy measurement; parametric downconversion

Conventionally defined, *radiometry* is the field that studies the measurement of electromagnetic radiation in terms of its power, spectral characteristics, and other parameters. The term applies to electromagnetic radiation characterization in a wavelength range from nanometers to tens of microns and at all optical power levels. Because radiometry is defined so broadly, a wide variety of measurement devices, or radiometers, with a variety of physical characteristics are used. It is therefore necessary to maintain a common scale for all radiometric measurements such that every family of radiometers can be traced to that scale.

Related to radiometry, the fundamental International system of units (SI) reserves a base unit for luminous intensity, called the candela. The art of luminous intensity measurement has evolved from comparison of various standardized candles and lamps prior to 1948 to the measurement of optical power suitable for cryogenic radiometers after 1979. Although measurement techniques are constantly refined and improved, the uncertainty of state of the art luminous intensity measurements is only 0.1%[1] and is in the order of 0.01 for radiometric measurements. This is the poorest accuracy of any SI base unit measurements. Hence, the search for methods to perform measurements with higher accuracy continues.

Advances in quantum optics, namely, in single- and bi-photon sources and single-photon detectors, have opened a new method for radiometry, which we will call 'quantum radiometry'. As we shall see, this designation is somewhat artificial and hence needs clarification. For the purposes of this review, quantum radiometry is defined as the measurement of electromagnetic radiation with the help of single-photon and photon-pair sources and single-photon or photon-number-resolving photodetectors. As such, these measurements are based on quantum mechanical laws that guarantee certain properties of photon statistics and rely on our ability to reliably detect single photons. Ideally, the accuracy of measurements based on photon counting scales as $1/N^{1/2}$, where N is the number of detected photons. Therefore, to match the state-of-the-art accuracy of conventional radiometry one needs to collect $>10^8$ single photon detections. It is feasible to acquire such a number in <100 s of measurement time, given the typical performance of modern single-photon detectors. It is also possible to achieve even lower uncertainties by measuring longer. Thus, 'quantum radiometry' can be considered as an alternative metrological technique to advance the accuracy of measurements of electromagnetic radiation.

1. Pre-history of quantum radiometry

The light sources that are used for what we call conventional radiometry are of course also based on the laws of quantum physics, for example, the blackbody. A blackbody is an object that absorbs all electromagnetic radiation that falls on it. By definition, the transmittance and reflectance of a blackbody equals zero. Because no electromagnetic radiation is reflected or transmitted, such an object appears black when it is cold ($T = 0$ K). A blackbody at temperature $T > 0$ K emits exactly the same radiance at exactly the same wavelengths that would be present in an environment at equilibrium at temperature T. It turns out that the radiation in such an environment has a spectrum that

*Corresponding author. Email: migdall@nist.gov

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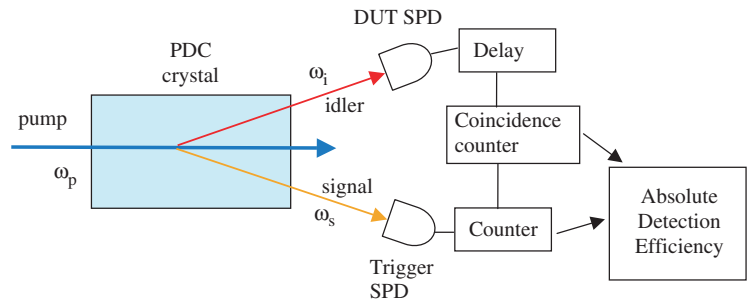

Figure 1. Detection efficiency measurement based on photon pair generation via parametric downconversion. (The color version of this figure is included in the online version of the journal.)

depends only on temperature. Gustav Kirchhoff introduced the term 'blackbody' and described this phenomenon in 1860. The derivation and physical understanding of blackbody radiation appeared 40 years later: on 14 December 1900 Planck presented his derivation of blackbody radiation laws based on the quantum hypothesis [2]. He proved that indeed, the properties of light emitted by a blackbody only depend on its temperature and several fundamental constants.

Such a direct and universal dependence between electromagnetic radiation and temperature makes a blackbody a good electromagnetic source standard for radiometry. That is, because the properties of the electromagnetic radiation can be fully described by just the temperature of an emitter, one can reproduce the source and independently measure (or calibrate) the response of radiometric equipment and hence independently reproduce a measurement scale. This is an example of a so-called fundamentally absolute calibration technique or 'primary standard method'. Because the blackbody radiation law derivation relies on the 'quantum hypothesis', a radiometric scale based on blackbody radiation is, strictly speaking, quantum. But note that while the derivation of this law requires quantum electrodynamics, most radiometric measurements of blackbody radiation in metrology do not rely on the discrete character of photons and can be processed using classical electromagnetic theory. Therefore, in this review we do not discuss blackbodies any further.

2. Basic theory

It turns out that laws of quantum mechanics offer a conceptually new primary standard method [3] for the calibration of single-photon detectors (SPDs). It follows from a quantum treatment of nonlinear optical phenomena that the signal and idler photons produced in a (spontaneous) parametric downconversion (PDC) process or in a four-wave mixing (FWM) process are correlated [4,5]. Naturally, this effect scales down to the single-photon level and allows the development of a conditional single-photon source [6]. That is, the presence of just one photon in a signal (idler) field heralds the presence of one and only one photon in the conjugate idler (signal) field (Figure 1). This remarkable property of PDC can be used to calibrate SPDs. This idea was further developed and put into practice [7] in the first calibration experiment using this technique, although the method had been used previously with other sources of photon pairs and before that for non-photonic particle detectors with two particle sources. We will consider this process in greater detail for PDC sources to show its relevance to metrology. The treatment of FWM sources is somewhat similar, although they have been much less explored for metrology applications and will be omitted from our discussion.

PDC occurs when photons from a pump beam propagate through a second order nonlinear crystal. Both energy and momentum conservation (also known as phase matching) conditions must be satisfied, that is, $\omega_p = \omega_s + \omega_i$, and $\boldsymbol{k}_p = \boldsymbol{k}_s + \boldsymbol{k}_i$, where ω refers to the optical frequency and $\boldsymbol{k}$ refers to the wavevector of the pump, 'p', signal, 's', and idler, 'i', photons. Therefore, conditioned upon a detection of a photon in one field $(\omega_i, \boldsymbol{k}_i)$, there must be a photon in a conjugate field $(\omega_s, \boldsymbol{k}_s)$. This effect can be used for calibration of SPDs. Namely, one field will be sent to a trigger SPD, and the other to the single-photon detector under test (DUT). The DUT must be positioned to collect all the photons correlated to those seen by the trigger detector. The DUT channel detection efficiency is the ratio of the number of coincidence events to the number of trigger detection events in a given time interval (assuming that the detectors only fire due to photons of a pair). A coincidence is defined as when both the trigger and the DUT detectors fire within a given time window due to detection of both photons of a downconverted pair. If we denote the detection efficiency of the DUT and trigger channels by η_{DUTchan} and η_{trigchan}, respectively, then the total number of trigger counts is

$$N_{\mathrm{trig}} = \eta_{\mathrm{trigchan}} N \tag{1}$$

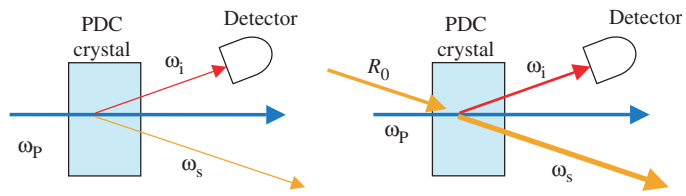

Figure 2. Calibration of spectral radiance setup with quantum zero-point fluctuations, left; and spectral radiance measurement, right. (The color version of this figure is included in the online version of the journal.)

and the total number of coincidence events is

$$N_c = \eta_{\mathrm{DUTchan}}\eta_{\mathrm{trigchan}}N, \tag{2}$$

where N is the total number of downconverted photons present in the trigger channel during the measurement period. The absolute detection efficiency of the DUT channel is

$$\eta_{\mathrm{DUTchan}} = N_c/N_{\mathrm{trig}}, \tag{3}$$

which is independent of η_{trigchan}. Thus, measurements of the trigger channel collection efficiency, and calibration of the trigger SPD are unnecessary, which justifies calling this a primary standard method. Note that η_{DUTchan} is the efficiency of the entire detection channel, including collection optics and filters, not just the efficiency of the DUT alone [8,9]. To determine the efficiency of the DUT alone, denoted by η_{DUT}, from η_{DUTchan}, all losses in the DUT path before reaching the DUT have to be determined. The total channel transmittance is the product of the transmittances of individual optical elements in the DUT path.

Because in general $\omega_s \neq \omega_i$, the spectrally selective components defining the calibration wavelength do not need to be in the DUT optical path when using the PDC-based detector calibration technique. Indeed, due to the energy conservation law $\omega_p = \omega_s + \omega_i$, all spectrally selectivity components can be placed in the trigger path. This conjugate frequency range can be thought of as a 'virtual' bandpass filter in the DUT path, defining the wavelengths that result in coincidences. This can be advantageous, for instance, when the detector to be calibrate is in the infrared, because one can do the spectral selection in the visible trigger channel for convenience [8].

Another metrological technique based on the quantum character of PDC uses quantum zero-point fluctuations in the vacuum field as a primary and omnipresent standard of spectral radiance. A source based on background fluctuations is a primary standard because its spectral radiance can be calculated from fundamental constants and does not require an external calibration. A radiance source under test (R_0) can be compared to this absolute standard, and such a comparison does not require intermediary transfer standards [10]. Indeed, the spontaneous decay of pump photons can be thought of as stimulated by a background due to zero-point vacuum field fluctuations, which is equivalent to one photon per mode. This one-photon-per-mode radiance can be written [4,10] in terms of wavelength λ, Planck's constant h and the speed of light c as hc^2/λ^5.

The measurement apparatus consists of an optical parametric amplifier pumped by a laser at ω_p. In the absence of R_0, the parametric amplifier generates pairs of photons consisting of one signal ω_s and one idler ω_i photon, due to spontaneous downconversion. A radiance source beam is directed into the parametric amplifier so as to overlap spatially and spectrally a particular output mode $(\omega_s, \boldsymbol{k}_s)$ (Figure 2). Photons from R_0 stimulate the downconversion process and hence increase the flux of idler photons in the conjugate channel. These photons are counted by a detector which may be an SPD. An extra advantage of this arrangement is that in general $\omega_s \neq \omega_i$ and the detection can be performed at the wavelength that is convenient for the observer, while the wavelength of the radiance source under measurement can be nearly arbitrary. Thus, it is possible to use the best detectors, currently Si detectors that operate best in the visible, to monitor, for instance, near- and mid-infrared bands of the electromagnetic spectrum where the measurement would otherwise be difficult.

To make an absolute measurement of radiance one simply needs to compare the single-photon flux in the idler channel with R_0 incident to that flux with R_0 blocked. Then the spectral radiance of the source under test in photons per mode is,

$$R(\omega_s) = \frac{1}{\eta_{\mathrm{system}}}\left[\frac{N_{\mathrm{on}}(\omega_i)}{N_{\mathrm{off}}(\omega_i)} - 1\right], \tag{4}$$

where N_{on}, N_{off} are the number of photons detected in the idler mode with the source to be measured on and off, respectively, and η_{system} is an overall system efficiency. Note that the detector does not have to be a photon counter. It could be any conventional detector as long as there is enough signal to noise to make a good measurement. For most practical applications, however, the photon flux is rather low, hence a photon counter is the best option. From (4), neither the idler detector nor the idler beam paths need

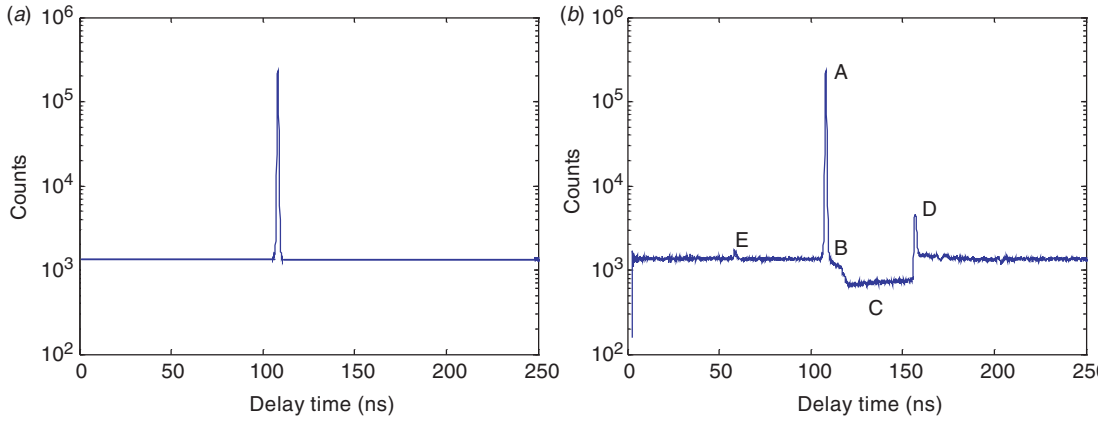

Figure 3. Correlation between photodetections in a PDC experiment (with a ≈100 ns delay in the DUT channel): (*a*) as seen by idealized SPDs with detection efficiency less than unity and (*b*) as seen with real SPDs with main features labeled. A – the main peak due to correlated photons, B – extended shoulder due to twilight events, C – the region where the detector is dead after firing, D – peak due to afterpulsing, E – minor correlated photons peak due to double back reflection in the trigger fiber and afterpulsing of the trigger APD. The broad background is due to the uncorrelated firing of the DUT. (The color version of this figure is included in the online version of the journal.)

calibration, because their efficiencies cancel. The only requirement is for the detector to be linear.

Similar to the detector efficiency method already described, some optical losses must be characterized. For this radiance measurement, the relevant losses are in the optical path from the source to the PDC crystal. Losses can be determined either by direct measurement or by calculation. In addition, an overlap between the beam under measurement and the volume of the nonlinear crystal that produces pairs must be determined and included in the overall efficiency η_{system}. Clearly, the system should to be designed in a way that maximizes this overlap to improve sensitivity and to reduce the uncertainty associated with its determination.

To determine the total overlap, both spatial and angular overlap must be considered. The spatial overlap can be found by integrating the product of the pump beam and R_0 within the crystal. All geometrical factors including aberrations of both pump and signal beams must be included. The angular factor provides the efficiency of the parametric process in the observation direction and is based on the phase matching and energy conservation constraints governing the process [11]. The first practical demonstration of this method was presented by Penin and co-workers in 1979 [12].

3. Practical considerations, implementation, and independent verification of metrological efforts based on primary standard methods

The basic theory discussed above assumes an ideal SPD. Namely, the SPD produces a count with a constant probability that only depends on detector efficiency and does not depend on any other factors; the detector should never produce a count if no photons were present; the SPD is assumed to be linear, etc.

It is expected that for the setup depicted in Figure 1, the correlation function would look as shown in Figure 3(*a*), if used with such ideal detectors. Here, the main correlation peak corresponds to the correlated signal due to PDC. It sits on a background that is generated by accidental or uncorrelated events, due to finite detection efficiency. However, in reality detectors suffer from multiple features that affect the correlation signal. Many of these features leave a trace on the correlation function as shown in Figure 3(*b*). With increasing accuracy of the correlated measurement, the accuracy of the background subtraction must also increase. To achieve that, all the features of SPDs must be well understood and characterized. A model of a typical silicon photon-counting avalanche photodiode that is useful for photon counting based metrology is developed in Ware et al. [13].

One example of a detector characteristic that must be dealt with is deadtime, which can be a particular problem for SPDs based on avalanche photodiodes. For these detectors, deadtime is defined as the time it takes to quench the avalanche in a p-n junction during which the SPD cannot detect a photon. This effect presents itself as feature C in Figure 3. Therefore, such SPDs are inherently nonlinear, because the detectors spend some fraction of measurement time in their 'dead' state, and that time is proportional to the observed count rate. Afterpulsing (feature D) occurs if occasionally a free carrier survives in the p-n junction region for the entire quenching time. If that is the case, the detector may produce a count even if no photon

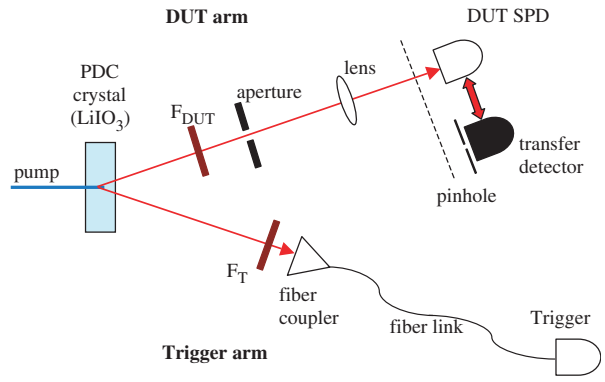

Figure 4. Setup for simultaneous calibration of a DUT with two independent primary standard methods. (The color version of this figure is included in the online version of the journal.)

was incident. There are other effects that can affect the single-photon detection process. Note that some of these features can be characterized with the help of the correlation function seen in Figure 3(*b*) [13]. To calibrate the DUT rather than the entire DUT channel, each optical surface in that channel must be independently characterized. Note that none of these effects requires an independent characterization of detection efficiency of a trigger detector. Nevertheless, some *a priori* knowledge of other characteristics of the trigger detector and the trigger channel are needed.

From Figure 3, we see that there are plenty of systematic effects that need to be considered and accounted for to make high accuracy measurements using a PDC setup and SPDs to turn these techniques into true metrology applications. With such a list of systematics to account for, it is important to independently verify the measurement results using a different technique to check for any systematic and implementation effects that may have been modelled incorrectly or are simply unknown to the measurer. A measurement protocol that yields independent calibration results using two different absolute calibration methods was proposed and implemented by Migdall et al. [14]. In addition to the correlated photon pair method just described, a conventional substitution method was used. This method relies on measuring the radiant power of the DUT channel with a transfer standard detector (traceable to NIST's detector radiant power scale) as well as with the photon-counting DUT [15,16].

For that comparison of the conventional and correlated methods, the experimental setup seen in Figure 4 was used. The signals from the Trigger and DUT SPDs are collected by a circuit that records both the overall number of trigger and DUT events, and the correlation between the trigger and DUT events in the form of a histogram with 0.1 ns temporal resolution. Because the coincidence events used for the two-photon calibration and the single-photon count rate of the DUT used for the conventional calibration are recorded simultaneously, the two types of calibrations are effectively made simultaneously. To complete the substitution method, the DUT SPD is replaced with a calibrated detector. To complete the comparison of the radiant power measured by the transfer standard detector to the number of counts measured by the SPD requires additional information about the spectrum of the source and specifics of the SPD itself.

4. Experimental progress

As mentioned, the first experiment using a PDC source to calibrate a SPD was reported by Burnham and Weinberg in 1970 [7]. They measured the detection efficiency of a photomultiplier tube using the correlation method and compared the value to a measurement made using a calibrated lamp source. Although they found a discrepancy of about 30% between the two measurements, they concluded that the two measurements were consistent within their estimated systematic uncertainty of 20%.

Eleven years later in 1981, Malygin et al. [17], independently calibrated PMT detectors, although their work was more of a demonstration effort than a metrological effort, as they did not report their uncertainties or give details about any comparison to an independent calibration technique.

In the late 1980s several groups began to look at SPD calibration using PDC. Bowman et al. [18] demonstrated the PDC calibration technique using avalanche photodiodes as the photon counting detectors in 1986 and reported an uncertainty of ≈10%. However, they did not make an independent comparison with a conventional standard. In 1987 Rarity et al. [19] performed a calibration using APDs, and validated their results by comparing the correlated photon calibration results to those obtained using a He–Ne laser attenuated with calibrated neutral density filters. Their reported uncertainty of ≈10%, i.e. the

uncertainty, was similar to that in [18]. This work was a great advance in single-photon metrology, because they carefully considered various components of uncertainties for photon counting systems that went beyond just statistical uncertainties. In 1991 Penin and Sergienko [20] reported a calibration of PMT detectors with a statistical uncertainty of 3%, but gave no further indication of measurement uncertainties. They also indicated that they compared the results to other reported conventional measurements, but gave no details on how these were obtained. In 1993 Ginzberg et al. [21] measured the efficiency of a PMT to an uncertainty of 10%, but made no comparison to a conventional standard. In 1994 Kwiat et al. [22] made a more careful study of the calibration technique and reported a 3% uncertainty for their detector efficiency value. They provided a detailed explanation of their method of accounting for uncertainties. In 1995 Migdall et al. [14] looked at several more effects. They performed a calibration of a PMT and compared the results to an analog detector calibrated against a radiometric standard at the National Institute of Standards and Technology (NIST). Multiple comparisons indicated that any measurement bias between the two methods was less than 0.6%, with an estimated uncertainty of 2% for individual detection efficiency measurements. Five years later, Brida et al. [23,24] performed another calibration, with a comparison against a Si trap detector for independent verification. In that work they included the collection optics and the spectral filter losses as part of the DUT (significantly simplifying the comparison of the two measurements), resulting in a 0.5% correlated photon calibration uncertainty and a 1% conventional calibration uncertainty for comparison. This result is the first successful attempt to cross the 1% threshold in reported uncertainties. In 2005, Ghazi-Bellouati et al. [25] performed a calibration of SPDs and reported uncertainties of a correlated method on the order of 1%. However, the verification uncertainty was 6.8%. In 2006, Wu and co-workers [26] calibrated an SPD with 2.1% uncertainty, but they did not report on an independent verification.

Finally, in 2006, Polyakov and Migdall [27] reported an individual measurement calibration uncertainty of 0.18% and overall verification uncertainty of 0.14%. As in [14], the correlated calibration was verified with a Si photodiode calibrated against a radiometric detector standard at NIST. This result was built on more than 10 years of experience in correlated photon metrology and used precision measurements available at NIST. This 2006 calibration and intercomparison effort and component analyses improved the understanding of experimental techniques associated with photon counting using SPDs and thereby allows the correlated photon calibration method to be used with confidence. The agreement between the two independent absolute calibration techniques verifies the model of SPD response to the same level of accuracy. On a negative side, a significant (at this level of uncertainty) nonuniformity of response across the SPD's detection area was revealed, making it difficult to use these particular SPDs for precision power measurements.

Table 1 summarizes the experimental results showing the general, but uneven trend from demonstration type measurements to more careful metrology efforts.

5. Single photon calibration technique dissemination

Based on the 2006 result, Polyakov and Migdall [28] concluded that while their experiment can be repeated in

Table 1. Uncertainty of two-photon method and its verification over time.

		Uncertainty of		
Year	1st author	Method	Verification	External comparison
1970	Burnham	≈20%	≈35%	Calibrated lamp
1981	Malygin	–		
1986	Bowman	≈10%		
1987	Rarity	≈10%		HeNe + attenuation
1991	Penin	>3%		
1993	Ginzburg	≈10%		Published values
1994	Kwiat	≈3%		
1995	Migdall	<2%	0.6%	Calibrated Si Detector
2000	Brida	≈0.5%	2%	Calibrated Si Detector
2005	Ghazi-Bellouati	1.1, 0.62%	6.8%	Cryoradiometer
2006	Wu	2.1%		
2006	Polyakov	0.18%	0.14%	Calibrated Si Detector

the laboratory setting, the calibration methods are not appropriate for everyday use. However, using photon-counting detector properties brought to light by their comparison of the two methods we were able to develop a relatively low cost calibration technique that will allow for 0.5% uncertainty. Hence, we proposed a portable, inexpensive and reliable scheme for calibration of SPDs. This scheme is based on a substitution method for the sake of simplicity. It turns out that to achieve an uncertainty of less than 1% in detection efficiency, additional measurements of DUT properties are required. In particular, based on their model, Polyakov and Migdall found that the complex afterpulsing behavior must be understood to allow for high-accuracy calibration. The following discusses some of aspects of this effect for avalanche photodiode SPDs.

For the case where an SPD is exposed to continuous wave (cw) light, in addition to the usual cause of an afterpulse due to lingering trapped carriers from a previous avalanche, an afterpulse can also result from a subsequent photon arriving during the last moments of the deadtime [13]. Thus, the afterpulse peak consists of photon-related afterpulses (or twilight counts) and ordinary afterpulses, not related to a photon absorption. Note that the ordinary afterpulse fraction is a fixed property of a specific SPD, in that it varies from unit to unit and does not depend on count rate, while the probability of getting a twilight event grows approximately linearly with increasing count rate. With count rates larger than ≈250 kHz, twilight counts noticeably affect the calibration result at the desired accuracy. Considering the high DUT count rates typically used in substitution calibration measurements, this property must be taken into account to obtain an accurate detection efficiency result.

By measuring the afterpulse fraction (defined as the likelihood of an afterpulse – not due to a second photon – given an initial count of the detector) at a range of DUT count rates, one can quantify and fit the linearly growing component of the count rate (see Figure 5). The slope of the resulting line is proportional to the duration of the twilight period and associated 'twilight detection efficiency' (which might not necessarily be a constant throughout this interval). In practice, this means that true detector deadtime can be shorter than the time of quenching, by several nanoseconds. That true detector deadtime must be measured to correctly convert a record of single-photon detection events into radiant power. This measurement requires simple additional electronics (AND gates, delay lines, etc.) and is described in [28].

To aid the effort for accessible high accuracy calibration, a set of 10 detector/amplifier packages will be calibrated at NIST to the accuracy of ≈0.3% and will be made available to the larger community.

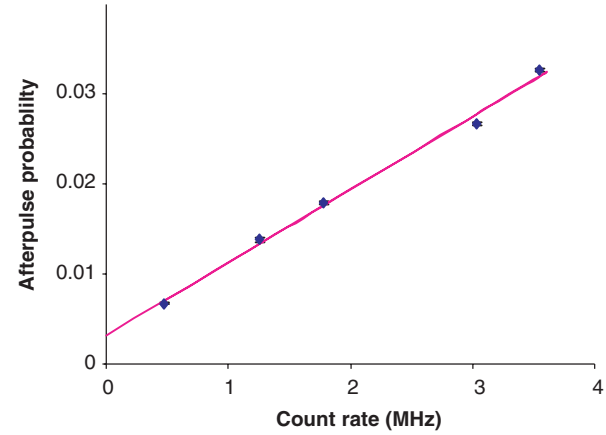

Figure 5. Afterpulsing of the detector under test APD at various count rates. Uncertainty bars are shown. (The color version of this figure is included in the online version of the journal.)

6. The future of quantum metrology

As the quality of single-photon detectors improves, the metrology based on single-photon detection will become more widespread and the accuracy must also improve. At the time of writing this review, the accuracy offered by quantum metrology is just about one order of magnitude worse than that of the conventional methods. Because of the progress in single photon detection technology, one might expect that the uncertainty of 'quantum' methods will become comparable (if not better) to that of the conventional methods in a decade or so. When that happens, an alternative absolute measurement technique will be at the disposal of the metrology community. The authors hope that their humble work will facilitate the progress of radiometry.

7. Conclusions

In conclusion, we have outlined the methods and reviewed efforts in the field of quantum radiometry.

Acknowledgements

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Note

1. Uncertainty values stated are standard uncertainties (k=1).

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